

# Starter

- a) Find the first three terms in the expansion in ascending powers of  $x$ . [3 marks]
- b) Using your expansion, approximate to four decimal places. [2 marks]

## N1

Understand and use simple, discrete probability distributions (calculation of mean and variance of discrete random variables is excluded), including the binomial distribution, as a model; calculate probabilities using the binomial distribution.

Students should:

- recognise when a situation may be modelled by a discrete random variable
- know and be able to use the fact that the sum of the probabilities of all possible outcomes of an event is 1
- understand a discrete probability distribution defined in a table or by a function
- be able to find the probability of a defined event in a given context
- recognise and be able to use  $B(n, p)$  as the notation for a binomial distribution with  $n$  independent trials where  $p$  is the probability of 'success' at any trial
- be able to state the conditions necessary for a binomial distribution and assess whether they are likely to be valid in a given situation
- be able to find the probability of an exact number of successes in a binomial distribution using the formula (given in the formulae book) or on a calculator
- be able to find cumulative probabilities in a binomial distribution
- calculate the mean, variance and standard deviation of a binomial distribution using the standard formulae given in the formulae book
- be able to use  $P(X \geq n) = 1 - P(X \leq n - 1)$  and similar results

Note: when using the binomial distribution, students must use a calculator to find probabilities.

# 10.2 Binomial Distribution

Some discrete random variables follow a **Binomial Distribution**.

Recap: From Chapter 2 on Binomial Expansions

If we have a total of objects where there are only two types of object, that is objects of one type and objects of another type, these can be arranged in different ways. This is also written as

$$\text{e.g. } {}^5C_2 = \frac{5!}{2!(5-2)!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 (3 \times 2 \times 1)} = \frac{60}{6} = 10$$

# 10.2 Binomial Distribution

So, if a trial results in either 'success' or 'failure' and you carry out trials, you can find the number of ways to arrange successes and failures using the binomial coefficient.

## **Example 1**

15 coins are thrown. How many ways are there to get 9 heads and 6 tails?



# 10.2 Binomial Distribution

Certain conditions must be met for a random variable to follow a binomial distribution:

1. Two possible outcomes in each trial ('success' and 'failure')
2. A fixed number of trials,  $n$ .
3. All trials are **independent**.
4. **Identical trials**, probability of 'success' ( $p$ ) is the **same** for each trial.

**YOU MUST LEARN ALL FOUR CONDITIONS!!**

# 10.2 Binomial Distribution

If  $X$  follows a binomial distribution, we write:

It has a probability function given by:

*[given on formula sheet]*

# 10.2 Binomial Distribution

Individual probabilities, , and **cumulative** probabilities, , can be found directly on a calculator.

If  $X$  can only take integer values,

and



# 10.2 Binomial Distribution

On the classwiz:

1. Menu, 7
2. For exact probabilities select 4: Binomial PD
3. Select 2: Variable
4. Input , ,
5. Press equals, check you get 0.0439...
6. Press menu, 7 again
7. Scroll down to 1: Binomial CD
8. Select 2: Variable
9. Keep the values the same, press equals
10. This time check you get 0.0546875

# 10.2 Binomial Distribution

## Example 2

- Decide whether each situation can be modelled using the binomial distribution. If it can't, say which of the conditions is not satisfied. If it can, find the required probability.
- a A fair die is rolled until it shows a six. Find the probability of getting two fours.
  - b Tom and Jerry play eight games of chess. The probability that Tom wins a game is 0.6, independently of any other game. Find the probability that Tom wins exactly four games.
  - c A student is trying to answer 20 quiz questions. The probability of getting the first question right is 0.9, but the probability halves for each subsequent question. What is the probability that he answers 10 questions correctly?
  - d In a particular village, 63% of five-year-olds attend the local primary school. What is the probability that, in a group of 15 friends, at least 10 attend that school?
  - e The probability that it rains on any particular day is 0.3. Assuming the days are independent, find the probability that it rains on more than four days in a week.

Not Binomial because it is not a fixed number of tri

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(c) Not Binomial as the probability of success is not constant.

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(d) Not Binomial, trials are not independent, if one child attends the school, their friends are likely to as well



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# 10.2 Binomial Distribution

## Example 3

Two fair six-sided dice are thrown 24 times.  $X$  represents the number of double sixes.

- a) Write down the probability distribution of  $X$  and its distribution function.

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- a) Write down the probability distribution of  $X$  and its distribution function.
- (success)(failure)

# 10.2 Binomial Distribution

## Example 3

Two fair six-sided dice are thrown 24 times.  $X$  represents the number of double sixes.

**b) Using the distribution function, find**

*[Using the calculator, use Binomial PD]*

# 10.2 Binomial Distribution

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Two fair six-sided dice are thrown 24 times.  $X$  represents the number of double sixes.

c) Find the value of

*[Using the calculator, use Binomial CD]*



# 10.2 Binomial Distribution

## Example 3

Two fair six-sided dice are thrown 24 times.  $X$  represents the number of double sixes.

d) Find the probability of **at least three** double sixes

*[Using the calculator, use Binomial CD]*

# 10.2 Binomial Distribution

## Example 4

If find

*[Using the calculator, use Binomial CD]*

# 10.2 Binomial Distribution

## Exercise 10.2A Fluency and skills

- 1 Given that  $X \sim B(5, 0.3)$ , find
  - a  $P(X=3)$
  - b  $P(X \leq 2)$
  - c  $P(X \neq 0)$
- 2  $X \sim B(8, 0.6)$ . Find, to 2 sf
  - a  $P(x \leq 0)$
  - b  $P(x \leq 3)$
  - c  $P(x < 5)$
  - d  $P(x > 2)$
- 3 The random variable  $T$  has a binomial distribution,  $n = 8$ ,  $p = \frac{1}{4}$ . Find, to 2 sf
  - a  $P(T=4)$
  - b  $P(T \geq 7)$
  - c  $P(3 \leq T < 5)$
- 4 Given that  $X \sim B(5, 0.4)$ 
  - a Write an expression for  $P(X=x)$
  - b Copy and complete the probability distribution table.

$x$	0	1	2	3	4	5
$P(X=x)$	0.078			0.230	0.077	0.010

- 5 A fair six-sided dice is thrown 4 times and the random variable  $X$  denotes the number of 6s obtained.
  - a Give the distribution of  $X$
  - b Find, giving your answers to 3 dp
    - i  $P(X=4)$
    - ii  $P(X > 2)$
    - iii  $P(1 \leq X < 3)$
- 6 A bag contains 12 counters. Three are red and the rest are black. A sample of five counters is taken, placing each back in the bag after it is chosen. Find the probability that the sample contains more than 3 red counters.

# 10.2 Binomial Distribution

## Exercise 10.2A Fluency and skills

1 a 0.132                      b 0.837                      c 0.832

2 a 0.00066                      b 0.17                      c 0.41                      d 0.95

3 a 0.087                      b 0.00038                      c 0.29

4 a  $P(X = x) = {}^5C_x \times 0.4^x \times 0.6^{5-x}$

b

$x$	0	1	2	3	4	5
$P(X = x)$	0.078	0.259	0.346	0.230	0.077	0.010

5 a  $X \sim B\left(4, \frac{1}{6}\right)$

b i 0.001                      ii 0.016                      iii 0.502

6 0.016 (to 3 dp)